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Encoding and Processing Spatio-Chromatic Image Information
using Complex Fourier Transform Methods

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Abstract

In this paper we develop a technique for encoding the variation of color over space using properties of the complex image Fourier transform where the image is defined by a complex number encoding, at each position, of hue (color angle) and saturation (color magnitude) values. The technique involves interpreting the complex coefficients of the non-symmetric spatio-chromatic frequency components in such a way as to generate spatial frequency gratings: basis functions which appear as “rainbow-gratings” which can directly encode the spatial juxtaposition of color. The transform can therefore extract spatio-chromatic structures or correlations in normal and iso-luminant images.

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1 Introduction

It has been noted for at least a century that human color perception is affected by the spatial distribution of color and not just by the local color (the chromaticity registered by each triple of cones at each point in the retinal image). Extended spatio-chromatic interactions are illustrated by artists' use of color contrast and in spatial induction experiments [1] and as exploited in the Retinex algorithm [2]. Chromatic appearance of test areas in colored Mondrians and in center-surround adaptation of the cones is apparently controlled, again, by color contrast [3]. These types of spatio-chromatic percepts seem to be cortical in origin as the receptive fields of cortical cells sensitive to color have spatially-mapped excitatory and inhibitory connections between centers and surrounds, each of which is sensitive to different regions of the visible spectrum [4].

On the other hand, in the color image processing literature the encoding of color has, until recently, also been limited to pixel-based analysis. However, over the past few years a number of authors have proposed models for the extraction of features which encode specific types of spatial color distributions and specific correlations of color, (for example, Caelli and Reye [5]), and for the extraction of color invariant to the illuminant [6]. Of more relevance to this paper is the recent use of Principal Components (Karhunen-Loeve decomposition) analysis of color images to form basic eigenimages corresponding to the predominant spatio-chromatic features in images (see, for example, Carevic and Caelli [7]). However, to this date, a *single* color image transform has not been proposed which is capable of encoding such spatio-chromatic properties via Fourier techniques. In the following we have developed such a representation and present some new types of spatio-chromatic frequency gratings which explicitly encode color gradients over images and are capable, using linear filters, of extracting chromatic edges even in iso-luminant images. It relies on the development of physical interpretations of *complex image* Fourier transforms and their associated grating modulations.

2 The Spatio-Chromatic Fourier Transform

The use of complex Fourier transforms is not new in computer vision. For example, Fourier descriptors have been used to describe features in the boundary of a shape encoded as $p(t) + iq(t)$ where t is a measure of distance around the boundary and (p, q) are image spatial coordinates [8]. Bonmasser and Schwartz [9] use a complex log-polar transform for size and rotation invariant pattern recognition. More recently, Thornton and Sangwine [10] have encoded hue and saturation as u and v values in complex Fourier space. They then argue for processing of this space which has the advantage of only requiring two FFTs to be performed (intensity and hue/saturation) over more traditional spaces, for example Red/Green/Blue space. They use the representation for color object location using phase correlation and cepstrum techniques. In our case, the Spatial Chromatic Fourier Transform (SCFT) is based upon a complex hue-saturation image format which is derived from the following transformations [11]¹:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412 & 0.358 & 0.180 \\ 0.213 & 0.715 & 0.072 \\ 0.019 & 0.119 & 0.950 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad \text{then} \quad \begin{aligned} Y &= Y \\ u &= \frac{4X}{X+15Y+3Z} \\ v &= \frac{9Y}{X+15Y+3Z} \end{aligned} \quad (1)$$

These values are then rotated by $\pi/30$ in order to align the u axis with red-green orientation and the v axis with the yellow-blue orientation in color space (see Figure 1a). Such chromaticity transforms and representations allow us to discriminate color differences independent of illumination levels. A 2D complex Fourier transform is then performed using the u and v components of Yuv as:

$$U(P, Q) + iV(P, Q) = \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} (u(p, q) + iv(p, q)) \exp[-2\pi i(Pp + Qq)/N]. \quad (2)$$

¹ITU-R Recommendation BT.709 — Basic parameter values for the HDTV standard for the studio and for international programme exchange (1990) — formally CCIR Rec. 709.

Here (p, q) correspond to the image spatial coordinates while $(u(p, q), v(p, q))$ represent the hue and saturation values at point (p, q) in the image. As usual (P, Q) correspond to the Fourier transform (spatial frequency) coordinates measured in picture cycles or cycles/pixel and the relative values of (P, Q) determine the spatial orientation of the frequency. The values of (U, V) correspond to the magnitudes of the frequencies which, in the complex case, describe how the color in the image changes spatially. What is novel here is the interpretation of the spatio-chromatic frequency gratings which result from this transform. Although a point in the two-dimensional frequency domain has a spatial frequency definition in terms of picture cycles, the actual *modulations* of such components are determined via the chromatic interpretation of the complex Fourier transform values at such points. Figures 1(b to d) show a symmetric pair of points in this frequency domain having complex values depicted by $U(P, Q), V(P, Q)$ and $U(-P, -Q)$ and $V(-P, -Q)$ and a (zero frequency) DC offset value. Figures 1(c and d) show the actual values of the components. It is important to note that *all* the components can be of different values for the same frequency (contrast this with the traditional use of Fourier analysis where the input data are real and the resultant frequency pairs are conjugated).

Interpreting how the values of $U(P, Q), V(P, Q), U(-P, -Q)$ and $V(-P, -Q)$ determine different color (hue) and saturation values over the spatial frequencies (P, Q) can be readily described as follows. The positive and negative spatial frequencies can be considered as identical spatial frequencies with modulations (angle indices) in opposite directions (clockwise and anti-clockwise). Each one sweeps out a circular path about the origin where the number of rotations corresponds to the spatial frequency e.g. lowest frequency means one rotation. The complex Fourier transform spatial frequency values $U(P, Q), V(P, Q)$ and $U(-P, -Q), V(-P, -Q)$ define the starting positions for these paths in color space as illustrated in Figures 2 to 4 for the clockwise (frequency (P, Q)) and anti-clockwise (frequency $(-P, -Q)$) cases. For the clockwise case in Figure 2a the start angle in (u, v) space is given

by $\theta = \tan^{-1}(V(P, Q)/U(P, Q))$ and the radius by $m = \sqrt{U(P, Q)^2 + V(P, Q)^2}$. Figure 2b shows a similar situation for the anti-clockwise case. The vector sum of the two circular paths produces the resultant path in color space. In effect the clockwise and anti-clockwise filters are configured to be of the same magnitude, and the phase of one the negative of the other. For example, the bold path along the u axis of Figure 2c is the result of summing the paths of Figure 2(a and b). Note this produces variation in color along the *real* axis starting at Red, moving through White to Green, and back through White to Red, only because the frequencies form a complex conjugate pair. Changing the values of U and V at P, Q and $-P, -Q$ with the constraint that they form a conjugate pair, will result in the same path but changes the phase or start point.

Generating color modulations corresponding to color changes along opponent color lines is of particular interest and is accomplished by setting the following values.

- For variation along Red–Green (u axis): $U(P, Q) = k$, $V(P, Q) = 0.0$, $U(-P, -Q) = k$, $V(-P, -Q) = 0.0$ where k is some chosen value dependent on the saturation of Red and Green (as shown in Figure 2).
- For variation along Blue–Yellow (v axis): $U(P, Q) = 0.0$, $V(P, Q) = k$, $U(-P, -Q) = 0.0$, $V(-P, -Q) = k$ where k is some chosen value dependent on the saturation of Blue and Yellow.

Examples of the two paths are shown as the axes in Figure 1a that intersect at the centre of the circle. In these cases, as the spatial frequency grating is generated over increments in angle, the color positions generated by the vector sum of the motions in clockwise and anti-clockwise directions result in straight line harmonic motion between opponent colors in (u, v) color as illustrated in Figure 2. Figure 3 shows the situation where the starting positions for the clockwise and anti-clockwise starting points have been shifted by $\pi/4$. The result is a straight line path between opponent colors at $\pi/4$ to the u and v axes.

Other paths through color space are possible. For example, changing the phase of one of

the waveforms, such as $U(P, Q) = k$, $V(P, Q) = 0.0$, $U(-P, -Q) = 0.0$, $V(-P, -Q) = 0.0$, causes a path through color space to be circular visiting all the different hues for constant saturation. Such a circular path is shown in Figure 1a. Another example is illustrated in Figure 4 where the clockwise and anti-clockwise magnitudes are different resulting in an elliptical path through color space. The two vectors are in the same direction along the leading diagonal defining the major axis and in opposing directions along the trailing diagonal defining the minor axis. The shape of the elliptical path is changed by varying the magnitudes m_1 and m_2 and the orientation by varying the two start angles S_1 and S_2 .

To illustrate the SCFT, Figure 5 shows images using single spatial frequencies of $P = 2$ and $Q = 2$ giving two cycles of color change along the p and q axes of the images with the main spatial frequency axis along the leading diagonal. Figures 5(a to c) shows the resultant images for just the clockwise path through color space with different phase angles i.e. different starting hues at $\pi/4$, 0 and $\pi/2$ respectively. Looking along the p axis, it can be seen that the same color pattern occurs but shifted based on phase. Figures 5(d to f) differ from Figures 5(a to c) in that the anti-clockwise case is shown. The results of straight line harmonic motion between opponent colors are sets of “rainbow gratings” as illustrated in Figure 5(g to i) based upon, in this example, the grating modulation ranges in opponent color coordinates. Figure 5g corresponds to Figure 3 and Figure 5h corresponds to Figure 2. The same translations occur because of the phase changes with the sequence of colors reversed. Finally Figures 5(h and i) show the results for paths between opponent colors along the Red-Green (u) axis and the Blue-Yellow (v) axes respectively. These are effectively combinations of Figures 5(b and e) and Figures 5(c and f) respectively. Figure 5g also shows a path between opponent colors although in this case the path is along the leading diagonal in uv space (at $\pi/4$ to the u axis). Again this is a combination of Figures 5(a and d). This path can be visualised by the reader in Figure 1a with the origin shifted down to the right and in Figure 3.

In all, then, the complete (hue, saturation) image is synthesised by sets of spatial frequency

gratings each of which is determined:

- *spatially* by frequency, orientation and phase, and
- *chromatically* by the four components of the two complex numbers corresponding to the positive and negative frequency values and resulting in the “rainbow” grating basis functions shown in Figure 5.

In order to further understand this encoding scheme, it may be helpful to imagine these basis functions (images) as a stack of “corrugated iron” sheets each with different frequency and phase characteristics, oriented in different directions and, most importantly, colored in different ways over the undulations. All these properties: undulations, orientations and coloring are determined via the complex Fourier transform and example gratings (single corrugated iron sheets) are shown in Figure 5. Such a representation allows us to investigate what a linear filter would correspond to in spectral and color image terms.

3 Spatio-Chromatic Filters

This transform now allows us to capture, *via linear operations*, color contrasts under normal and iso-luminance conditions, oriented or not, and a wide range of spatio-chromatic features previously encoded by more complex non-linear processes. However, it is important to interpret the notion of “filtering” or “frequency modulations” in this case. First, since each frequency component is complex and both positive and negative frequencies need to be considered together, a new variety of filters emerge. The simplest version is one based upon the Gaussian modulation of the frequency magnitudes determined, for example, in terms of the saturation of the sets of opponent color components. This corresponds to determining the saturation of given Red-Green and Blue-Yellow colors at each spatial frequency; but, in general, this “desaturating filter” is not limited to the opponent color example and would be

typically defined as a modulation of:

$$\text{Amp}^2(P, Q) = U^2(P, Q) + V^2(P, Q) + U^2(-P, -Q) + V^2(-P, -Q). \quad (3)$$

For example, consider the spatially-oriented Gaussian (so-called ‘‘Gabor’’ filter) as illustrated in Figure 6b in Fourier space and defined by:

$$\begin{aligned} G(P, Q) = & \exp \left[\frac{-1}{2} \left(\frac{(P - P_0)^2}{\sigma_P^2} + \frac{(Q - Q_0)^2}{\sigma_Q^2} \right) \right] \\ & + \exp \left[\frac{-1}{2} \left(\frac{(P + P_0)^2}{\sigma_P^2} + \frac{(Q + Q_0)^2}{\sigma_Q^2} \right) \right], \end{aligned} \quad (4)$$

for filter center (P_0, Q_0) and isotropic bandwidth defined by σ , and where its orientation (which is variable from 0 to 2π radians) is along the U axis. This filter, when used to modulate $\text{Amp}(P, Q)$ to result in:

$$\text{Amp}'(P, Q) = \text{Amp}(P, Q) \times G(P, Q), \quad (5)$$

desaturates all the colors at a given spatial frequency as a function of the Gaussian modulation function with peaks at (P_0, Q_0) and $(-P_0, -Q_0)$. The resultant inverse Fourier transform would only retain the *color distributions* of those spatial frequency components near this center frequency, and defined by:

$$u(p, q) + iv(p, q) = \sum_{P=0}^{N-1} \sum_{Q=0}^{N-1} (U(P, Q) + iV(P, Q)) \exp[2\pi i(Pp + Qq)/N] \quad (6)$$

where

$$F(P, Q) = \text{Amp}'(P, Q)(\cos \theta + i \sin \theta + \cos \phi + i \sin \phi) \quad (7)$$

for $\theta = \tan^{-1} V(P, Q)/U(P, Q)$ and $\phi = \tan^{-1} V(-P, -Q)/U(-P, -Q)$.

The resultant filtered image must then be rotated back to correct Yuv space and converted

from Yuv to XYZ and from XYZ to RGB using:

$$\begin{aligned} X &= \frac{3Y}{2} \frac{u}{v} \\ Y &= Y \\ Z &= \frac{4X}{3u} - \frac{X}{3} - \frac{5}{Y} \end{aligned} \quad \text{then} \quad \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.241 & -1.537 & -0.499 \\ -0.969 & 1.876 & 0.042 \\ 0.019 & -0.204 & 1.057 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}. \quad (8)$$

Figure 6a shows a Gabor lowpass filter with the center at the DC or origin position which attenuates all high frequencies with a higher cut off in the p direction than in the q direction. Figure 6b shows a Gabor bandpass filter designed to pass only frequencies in the general p direction. Note that for both filters, both the real and imaginary components of the +ve and -ve frequencies, (P, Q) and $(-P, -Q)$, are processed identically using the same filter. Figure 6c shows an iso-luminant image where the different pixel values are only apparent in the hue and saturation values. Figure 6d shows the result of processing using the Gabor filter of Figure 6a. As expected the edges have been blurred. For example, consider the polygon in the bottom left corner of the image, the sharp transition between blue and red at the edges is now a smooth transition through the desaturated colors between blue and red. Note there is a slight color shift between the two images because of small inaccuracies in the color transformations and some scaling in the Gabor filtering. Figure 6e shows the enhanced result of the filter of Figure 6b. As expected low and high pass frequencies have been attenuated in the p direction and all frequencies attenuated in the q direction. The image is enhanced because the amplitude of the colors is very low and difficult to assess. Note the color transitions occurring at the edges, especially in the lower left corner (magnified in Figure 6f). The bandpass filter has resulted in oscillations between red and blue at the edges and little change in color elsewhere. Again the slight blue-ish tint to the image is due to inaccuracies in the color transformations.

Overall, Figures 6(c to f) illustrate how such linear filters can easily extract edges and, in general, color contrasts, in images *without relying on luminance nor non-linear processes*.

Equally, isotropic Gaussian filters in this spatio-chromatic transform domain would encode center-surround opponent color detectors — also known to occur in the vertebrate visual system (see Livingstone and Hubel [4]). Note that the spatial frequency image of the filter indicates how color *saturation* is being modulated over spatial frequencies. Other forms of filtering also exist. For example, the desaturation-filters can be separately applied to each opponent color scheme or selective color ranges. Color or hue filters can also be defined as a function of spatial frequency (P, Q) by defining separate functions for $U(P, Q)$, $V(P, Q)$, $U(-P, -Q)$ and $V(-P, -Q)$. Bandreject filters are possible by using the filter $1 - G(P, Q)$ where $G(P, Q)$ is defined in equation 4.

4 Conclusions

We have investigated a new method for encoding the spatial distributions of color over space using a single complex Fourier transform. The technique hinges on the interpretation of the complex components of the frequency components and the mapping of frequency modulations into paths in color space. Results demonstrate how spatio-chromatic correlations and features can be readily extracted using linear filters and also demonstrate that spatially color-opponent filters (receptive fields) can be encapsulated by simple linear filters in this new spatio-chromatic Fourier transform domain. An issue that needs to be considered is the choice of color space. From Figure 1a it can be seen that not all colors are representable spatio-chromatically. Other color spaces such as $L^*u^*v^*$ (also known as CIELUV) [11] may be more suitable.

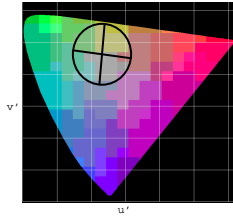
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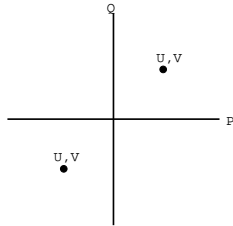
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5 Acknowledgements

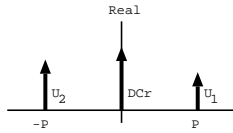
This research was partly funded by a large grant from the Australian Research Council and from a Generic Industrial Research Development Grant from DEETYA.



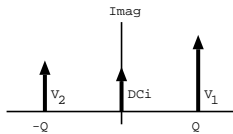
(a) CIE chromaticity diagram with axes overlaid.



(b) Frequency space axis.

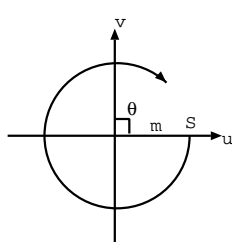


(c) Real components and DC offset.

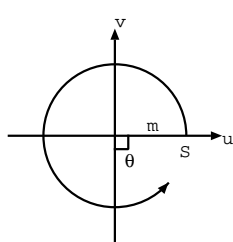


(d) Imaginary components and DC offset.

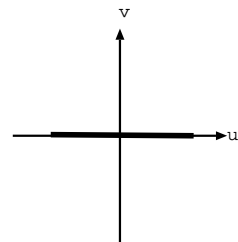
Figure 1: Frequency-space diagrams.



(a)



(b)



(c)

Figure 2: Phasor diagrams for travelling through color space along the u axis showing (a) clockwise path through color space, (b) anti-clockwise path through color space, and (c) vector sum of (a) and (b) showing resultant straight line path through color space.

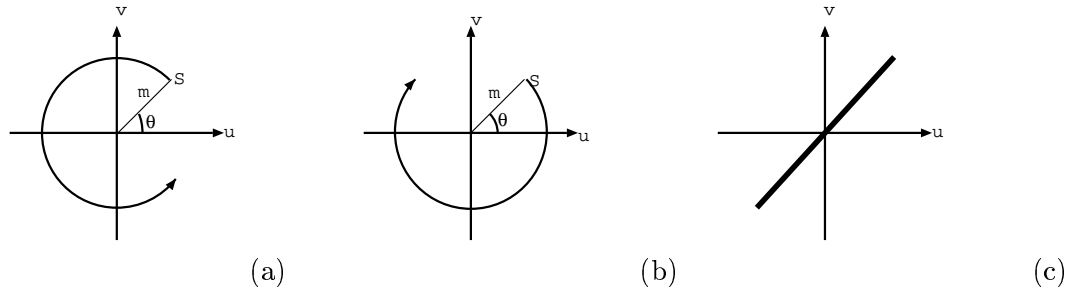


Figure 3: Phasor diagrams for travelling through color space along a line through opponent colors via the origin showing (a) clockwise path through color space, (b) anti-clockwise path through color space, and (c) vector sum of (a) and (b) showing resultant straight line path through color space.

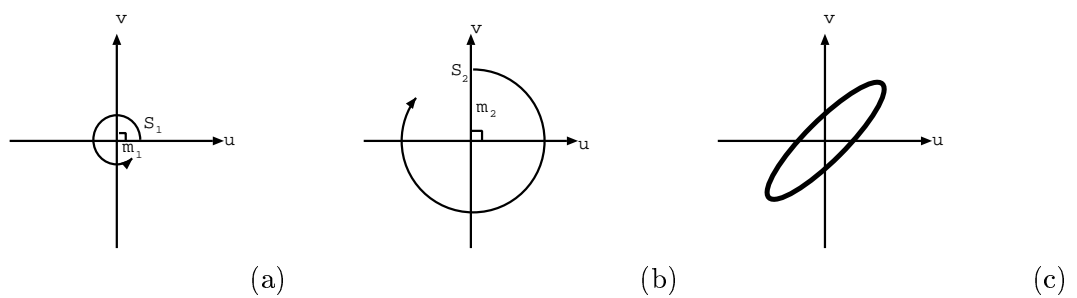


Figure 4: Phasor diagrams for travelling through color space along an elliptical path showing (a) clockwise path through color space, (b) anti-clockwise path through color space, and (c) vector sum of (a) and (b) showing resultant elliptical path through color space.

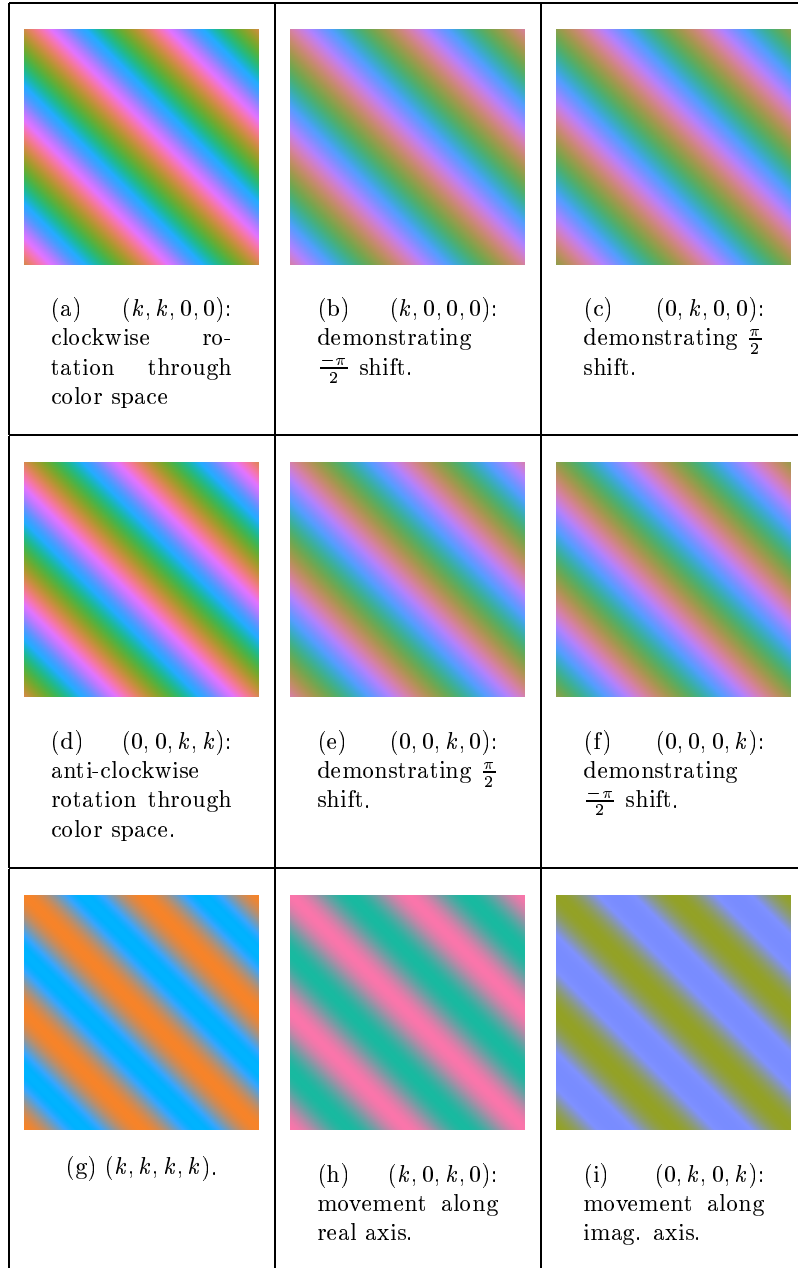


Figure 5: Spatio-chromatic frequency gratings for different combinations of $U(P, Q)$, $V(P, Q)$, $U(-P, -Q)$ and $V(-P, -Q)$.

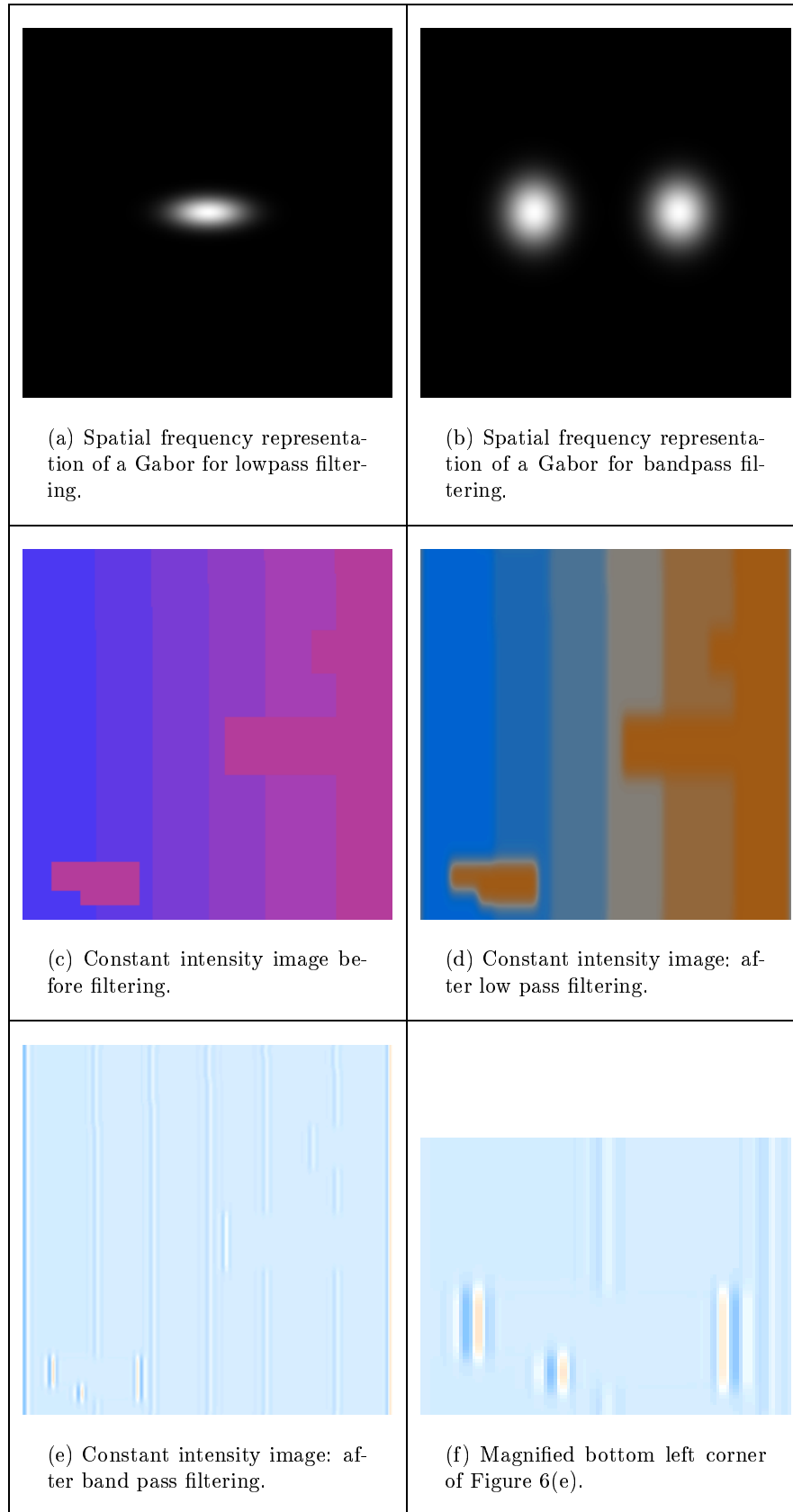


Figure 6: Basic iso-luminant edge filtering.